The *Real* Facts of Life

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The problems in Harold Walker's latest essay, amusingly titled "The Facts of Life", start with his very first line:

> Digital Modulation is usually a form of pulse modulation, of which, there are two types-Pulse Width and Pulse position.

What about pulse amplitude modulation (PAM), phase shift keying (PSK) and frequency shift keying (FSK), just to mention a few others?

> All pulses are analyzed by means of their Fourier Transform.

Yes, they certainly can be. It's too bad that Walker doesn't know how to do it.

> The transform consists of two parts: 1) a series of frequencies, that when added will create the original pulse shape. 2) A change in amplitude that varies with the pulse width or pulse spacing. The two are seen in Equation 1. Figure 1 shows a series of square wave pulses.

Walker is thoroughly confused. The continuous Fourier transform consists of a single integral:

$$X(f) = \int_{-\infty}^{+\infty} x(t)e^{-i2\pi ft}dt \tag{1}$$

To evaluate the spectrum of $x(t)$ at any particular frequency f, you plug them into this integral and integrate over all time. $x(t)$ can be any function; unlike the discrete Fourier series analysis, $x(t)$ is not assumed to repeat over any period.

The inverse Fourier transform, which converts the frequency spectrum $X(f)$ back into the time domain, is

$$x(t) = \int_{-\infty}^{+\infty} X(f)e^{i2\pi ft}df \tag{2}$$

Note that the value of $X(f)$ depends on $x(t)$ over all time, and the value of $x(t)$ depends on $X(f)$ over all frequencies.

Figure 1 shows a series of square wave pulses. The Fourier transform of a square wave pulse is a sinc, or sinx/x function, shown in Fig. 2.

There are several things very wrong with this picture. For one thing, the Fourier transform of a time-domain signal must be in the frequency domain, but figure 2 has *time* on the horizontal axis! Second, the picture doesn't look anything like the Fourier transform of figure 1. The Fourier transform of a *single* positive-going pulse with amplitude A_m starting at $-\tau/2$ and ending at $+\tau/2$ ($\tau > 0$) is actually

$$A_m \tau sinc(f\tau) \tag{3}$$

This is a continuous function in frequency; i.e., a single square pulse spreads its energy over many adjacent frequencies.

However, the Fourier transform of the *infinite* repeating train of *identical* pulses shown in Fig 1 is a different story:

$$A_m \frac{\tau}{T} sinc(f\tau)\delta(f - n/T) \tag{4}$$

where n is any integer and $\delta()$ is the Dirac delta function, defined as

$$\delta(x) = \infty, x = 0 \tag{5}$$

$$\delta(x) = 0, x \neq 0 \tag{6}$$

$$\int_{-\infty}^{+\infty} \delta(x)dx = 1 \tag{7}$$

Equation (4) describes a series of infinitely narrow *spectral lines* at discrete frequencies that are integer multiples of $1/T$. There is no energy at any frequency that is not an exact multiple of $1/T$.

Line spectra are characteristic of infinitely repeating functions. Non-repeating functions (e.g., random data) do *not* have line spectra; their energy is spread out over infinitely many frequencies.

This is directly relevant to Walker's VMSK delusions. VMSK is actually the sum of a repeating function and a non-repeating function. The repeating clock waveform is responsible for that conspicuous spectral line, and the non-repeating random data is responsible for the wideband "grass".

Figure 2 appears to have been copied from a textbook discussion of the Nyquist Theorem. It actually shows the *time-domain* sinc pulses produced by running time-domain impulses (not square pulses) occurring every T_b seconds through an ideal low-pass filter with cutoff frequency $\frac{1}{2T_b}$ Hz. Note that when one sinc function reaches its peak, the others are all zero. This is the zero-ISI (intersymbol interference) condition, and this is impossible with any filter narrower than $\frac{1}{2T_b}$. This is another basic fact of life that Walker refuses to accept.

If the pulses shown in Fig. 1 are used for modulation (representing a 1,1,1,1,1 pattern in pulse position with AM modulation using on/off

> keying), the result will be a series of pulses as shown in Fig. 2. They will overlap because the sinx/x shape is broader than the square wave pulse.

If the pulses shown in Fig 1 are sent down a channel, their appearance at the receiver critically depends on the frequency and phase response of the channel and the transmit and receive filters. You'll only get the ideal sinc pulses shown in Fig 2 when the concatenation of the original pulse spectrum with all the filters and the channel response yields an ideal brick-wall low pass response with constant group delay (linear phase) and cutoff at $\frac{1}{2T_b}$ Hz.

The bad news: it's impossible to build such ideal filters. The good news: it's not actually necessary to do so. *Any* pulse shape that goes to zero at the center of every adjacent bit will suffice; it doesn't actually have to be the sinc function.

It is still necessary that the half-amplitude (-6dB) bandwidth be no less than $\frac{1}{2T_b}$ and the response be linear phase. But the combined response need not be "brick wall" as long as it is anti-symmetric around $\frac{1}{2T_b}$. "Raised-cosine" filter responses are a popular way to implement these responses in practice.

> Suppose instead of on/off keying, the pulses in Fig. 1 are offset in DC level so that the time spent above and below the center line (0 volts) is equal, as in BPSK modulation transmitting a 101010101 pattern, then the sinx/x pulses seen in Fig. 2 are the same, but the polarity is reversed on alternate pulses. This is seen in Figs. 3 on the center line. The pulses after reversing on alternate pulses creates the equivalent of the sinx/x) 2 pulses seen in Fig. 4.

No, if we remove the DC offset from the pulse train of Fig 1, the only change is to remove the spectral line component at zero frequency. If we're going to look at this new "balanced" signal as an alternating 1010... pattern, we must double the data rate and increase the pulse duty cycle $\frac{\tau}{T}$ to unity. The Fourier transform remains a series of spectral lines, although the one at zero frequency is now gone.

> Fig. 4 Sinx/x pulses when the voltages are added. This is the detected pattern when using an XOR gate as detector with a reinserted carrier.

This quote and the accompanying Fig 4 which appears to be hand-drawn by Walker, simply make no sense. Fig 4 doesn't even depict a *function*, which by definition must have exactly one value at any given time.

> Equally spaced pulses of equal width carry no modulation.

It would be more correct to say "Equally spaced pulses of equal width carry no *information*".

> By using pulse positioning of the pulses, ones and zeros can be added. In Fig. 3, a one is transmitted when the pulse is early and a zero if the pulse is delayed. This creates the pulse position modulation shown in Fig. 3, at the bottom. The spectrum of these pulses shows several things that are important to an understanding and analysis of the pulse position method. When alternating ones and zeros are transmitted, there is a difference in time periods that creates a form of pulse width modulation, with the pulse widths being end to end and of reversed polarity.

Perhaps, but this is irrelevant. There's no information in a completely predictable pulse train. Because the pulse train repeats, its spectrum consists of one or more spectral lines. The fact that filters might remove every spectral line but one proves absolutely nothing – except that the signal conveys no information.

If the pulse train consisted of actual (i.e., random) data, then its spectrum would no longer consist entirely of spectral lines. There'd be a broad spectrum of "grass", and it would not be possible to filter it down to a single spectral line without making it impossible to recover the data.

> Fig. 5. The spectrum of the unmodulated signal transmitting 101010101 using BPSK modulation is shown in Fig. 5. The pulses of equal width and time spacing create a very widespread signal containing a great number of pulses. The two nearest the center are important, the remainder can be filtered off.

This actually looks correct. Yes, a BPSK modulator given a repeating 101010... data sequence with proper filtering either at baseband or after modulation will produce a spectrum consisting of two spectral lines, one at $f_c - \frac{f_r}{2}$ and the other at $f_c + \frac{f_r}{2}$, where f_c is the carrier frequency and f_r is the bit rate. But has been already stated, there is no information in such a sequence, so this is irrelevant.

Walker's inverse Fourier series in his Equation 1 is *complete* garbage.

For starters, there appears to be a spectral term $4A_{av}$ that is constant for all frequencies. That implies that the time domain representation of VMSK has a delta function at time zero. Where did *that* come from? Second, his definition of θ is utter gibberish. At first it doesn't include the frequency f; then for a moment it does; and in the next moment it's gone again. By definition, the inverse Fourier series is a sum performed over all frequencies. Yet here, f is not to be found.

This is not the first time that Walker has confused the frequency and time domain representations of a signal.

But even if his discrete inverse Fourier series were correctly written, its use here would still be incorrect. Unlike the continuous Fourier transform, which works for any arbitrary function, the discrete Fourier series assumes that the time-domain function repeats forever with some period T. This may be true for an artificial data sequence like 101010... but it is *not* true for random user data.

A *random* data sequence will produce a very different spectrum over a continuous, broad range of frequencies. Depending on the filtering, this continuous spectrum will extend down to at least $f_c - \frac{f_r}{2}$ and up to at least $f_c + \frac{f_r}{2}$. This is the minimum Nyquist bandwidth required to carry a binary data stream at a rate of f_r.

Again, Walker's analysis on page 3 is utter garbage.

> Figure 6 shows the effect of using the pulse modulation of Fig. 3. A low level hump appears around the center, or carrier frequency. This is referred to as 'grass'. It is a noise characteristic that must be reduced to satisfy the FCC. It arises from changes in Aav. (To be discussed later). Figure 6 is without any bandpass filtering.

No, the so-called "grass" is *not* "noise". It's the *data*! Why else would it appear only when random data is sent? What other noise-like sources are there in the system *besides* the randomness of the data? If the data rate is high, the required bandwidth will also be high, and the power outside the nominal channel bandwidth may have to be "reduced to satisfy the FCC". But it cannot be eliminated, and even reducing it necessarily reduces the signal power the receiver needs to recover the data in the presence of noise.

Walker's "changes in A_{av}" are nothing more than the data itself. They cannot be decreased without impairing modem operation.

> When the pulse delay is less than 1/5 of the pulse width, the spectrum appears as a series of individual frequencies. This is accompanied by the grass, which has a level related to the delay period. If the delay period is 1/20 the bit period, the grass is at -40 dB in a 3 kHz filter bandwidth. (Analyzed later). Using pulse width or pulse delay modulation, results in 'Coded BPSK', which has the same Bit Error Rate for a given C/N as ordinary BPSK.

The only reason the spectrum "appears as a series of individual frequencies" as the pulse delay is decreased is because the data-carrying "grass" becomes too wide and too thin to show up readily on a spectrum analyzer; the display is swamped by the strong, narrow clock. It's like the solar corona: too dim and diffuse to see from earth except during a total eclipse, but it's still there.

So is the grass in VMSK. If we exclude the power wasted on the clock (the strong spectral line), then yes, the bit error rate (BER) for a given C/N will be the same as BPSK because the signal *is* BPSK. However, if we include the power wasted on the clock, the overall performance will suffer greatly because the clock takes up more of the total transmitter power. This is easily verified with computer simulation.

> Figure 7 shows how filtering is used at RF with BPSK modulation to remove the spectral spread. A raised Cosine filter can reduce the harmonics to below 50 dB peak values. The straight lines drawn in the center of the BPSK arches are for Coded BPSK, where the

harmonics are single frequency lines. Filtering removes or reduces the 3 rd , 5 th , etc harmonics, but leaves the fundamental frequency of the Fourier series at full strength.

The “straight lines” drawn here for the VMSK spectrum merely represent the clock and its harmonics. The diffuse, broadband grass that actually carries the data is much lower, so it isn't as apparent. But it is most assuredly still there.

Filtering can also be done at baseband. This is a common practice for GMSK modulation, which also limits the spectral spread. Figure 8 shows how a low pass filter is used to remove the harmonics at baseband. This has an advantage in some cases in that the filtering can be done with a DSP or FPGA chip instead of with a crystal.

Once again, it does not matter whether the filtering is performed at RF or at baseband. So the same problems with Fig 7 are present in Fig 8; only the VMSK clock components are drawn, but the data is present in the broadband “grass”.

Fiure 9 shows the pulse position modulated signal using the encoding method of Fig. 3, after baseband filtering. The harmonics are removed so that only the fundamental of the modulating frequency remains. The spectrum is shown in Fig. 10 for 'Coded BPSK'. The time differences are retained, even though the spectrum appears as a single line, if the time differences are small.

The “harmonics” here are at the multiples of the baseband pulse rate. These can indeed be safely removed; that's why the zero crossings shown on the scope are gradual rather than abrupt. But without the broadband grass within the Nyquist limit of $f_r/2$, the open “eye” seen at the baseline would close. And with the eye closed, the receiver could no longer distinguish a 0 from a 1.

Note how tiny the eye opening is compared to the amplitude of the pulses. That's because so little of the VMSK signal power is in the data-carrying grass; most of it is wasted on the clock, which is the same for both 0s and 1s.

Figure 10. The baseband spectrum of 'Coded BPSK” using the method of Fig. 3 and the filtering of Fig.8 . Random data causes the grass level seen here to be more than 40 dB below the data spike. This is random noise that must be further reduced by means of a special very narrow band filter at RF.

Finally, we see the grass. But at the risk of being overly repetitive, the grass is not random noise. It comes directly from the data's randomness, and it is the *only* VMSK signal component that distinguishes the 0s from the 1s.

Remember in our discussion of the Fourier transform that the value of the time-domain function $x(t)$ at any time t depends on $X(f)$ over all frequencies.

That's why the seemingly insignificant grass, far from being incoherent noise, is so vitally important. Every tiny little component of grass at frequency f has a phase such that when the inverse Fourier integral is evaluated for a time t within the narrow window in the center of the bit, the grass components all add in phase, pushing the transition a little to the left or to the right depending on whether a 0 or a 1 was sent. Without the grass, this would not be possible.

Yes, the grass *is* weak. That's the reason the eye opening is so small. In a proper suppressed-carrier BPSK system, where *all* of the transmitter energy is spent on the data-bearing "grass", the eye opening is much larger. And such a system would perform much better than VMSK in the presence of noise.

> Figure 11. The RF spectrum of Fig. 10, transmitted with suppressed carrier, using the baseband signal of Fig. 10. The grass appears higher because the spectrum analyzer filter. BW is 3 kHz instead of 300 Hz. This is characteristic of noise, where the power level varies directly with bandwidth. The bandwidth of the spectrum analyzer filter has little effect on a signal bearing spike.

This is utterly bogus. A spectrum analyzer simply responds to the total signal power within its resolution bandwidth. It doesn't matter whether the signal in this bandwidth consists of one big discrete spectral line, several spectral lines, or a piece of a much wider broadband signal. It simply sums up the total in-band power and displays the result. So Walker has it exactly wrong: the displayed amplitude will not vary with resolution bandwidth when the input signal consists solely of a non-information-bearing spectral line centered at the analyzer's current frequency, but it *will* vary directly with bandwidth when the input signal is wider than the largest resolution bandwidth in use.

A broadband signal can represent either noise or data, so it may or may not represent useful information. But a spectral line – be it an AM broadcast station carrier or the VMSK clock – can *never* carry useful information.

Take a look at an AM broadcast station with a spectrum analyzer. If Walker were right, you might conclude that all of the information is in the carrier and the surrounding sidebands are merely "noise".

> When a carrier is reinserted at the detector, the sine wave of Fig. 10 is restored. If this is squared up, the result is the waveshape in Fig. 3. See other papers regarding the use of the sideband alone as a reference instead of the carrier.

Throughout his VMSK writings, Walker makes a big deal about the ability to demodulate VMSK without a carrier. But that's not really right; the strong VMSK clock is really just a carrier that has been offset by the data rate f_r from the RF carrier f_c. It's well known, especially to radio amateurs who have used this technique for years, that when you create a modulated signal at some carrier frequency f_{sc} and then pass it through an upper-sideband SSB transmitter tuned to a carrier frequency f_c, the resulting RF signal is identical in every respect to one created by direct modulation of an RF carrier at $f_c + f_{sc}$.

If the SSB transmitter operates in LSB mode, the equivalent RF carrier will be $f_c - f_{sc}$ and the modulated spectrum will be inverted in frequency. With VMSK, $f_{sc} = f_r$, i.e., the subcarrier frequency is equal to the data rate.

> Some critics object to the claim that the bandwidth transmitted is 1 Hz wide. Actually that is all that needs to be transmitted. The bandwidth for Shannon's Limit and other considerations is still from carrier to sideband- the full Nyquist bandwidth. Using FCC measurement standards, the bandwidth as transmitted SSB-SC is 1 Hz wide. This is seen above.

"Critics object" because this claim is simply and provably false. Walker's analysis is fatally flawed because he assumed an information-free 101010... data sequence. The required RF bandwidth for any signal that has been passed through a single-sideband transmitter is exactly equal to that signal's baseband bandwidth.

As has been proven many times, the only reason VMSK appears so narrow is that the strong, information-free spectral line at $f_c + f_r$ completely dominates the diffuse, broadband modulation spectrum ("grass") that carries the actual data. With various baseband coding tricks the grass can be shaped or pushed around in frequency but it can never be eliminated without destroying the ability to demodulate it.

Such tricks are clearly behind Walker's recent claims that he's eliminated the grass by manipulating A_{av}. Anyone who understands Fourier transform theory will know that this merely a shell game that Walker can never win. He may be able to fool a few humans (including himself) but he will never be able to fool nature.

The remainder of Walker's document talks about meeting FCC emission masks. As explained in my previous analyses, the FCC emission masks denote an artificial definition of "bandwidth" that can be much less than the true bandwidth required by the Nyquist and Shannon theorems. So the fact that VMSK may meet an FCC mask is irrelevant in actually making it work in a narrow bandwidth with many other signals packed close by. This is something that Walker has never demonstrated.

VMSK remains worthless, and Walker remains a moron incapable of learning from his mistakes.